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by Robert P. Gilles

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October, 1989

Equilibrium in a Pure Exchange Economy with an Arbitrary Communication Structure[1]

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Abstract

In this paper we develop a two stage general equilibrium model of a socially structured market with pure trade only. The model incorporates noncooperative as well as cooperative trading processes. Both kinds of trading processes are based on two natural consequences of limited trade possibilities among the participants on the market.

Firstly, limited trading possibilities create natural social asymmetries of the agents in the market. These asymmetries result in a hierarchical system of leadership or dominance in the market structure. The agents in the higher echelons in the hierarchy on the market set prices for the traders in the lower echelons. Hence, higher echelons consist of price setters, while the agents at the lower echelons act as price takers. This describes an extension of the famous Stackelberg leadership in oligopolistic markets.

Secondly, limited trading possibilities of the agents on the market generate asymmetries in information on the prices which are quoted by the price setters in the hierarchically structured market. The informationally asymmetric agents induce re-trading or re-contracting processes outside the official institutionalized market structure as described by the hierarchical system of price setters. The re-trading processes on the market are described with the use of a cooperative re-contracting configuration. It is shown that these cooperative re-trading processes cause the prices, which are quoted within the hierarchically organized market, to be consistent with respect to the cooperative re-contracting principle. Thus, in the equilibrium that is reached no allowable coalition has the power as well as the need to re-trade, given the prices as quoted by the price setters and the proposed allocation.

1 Introduction

This paper is devoted to the development of a game theoretic model of a pure exchange or trade economy with limited possibilities to trade and to communicate. Our point of departure is the traditional setting of general equilibrium theory as discussed and developed rigourously by Debreu (1959). Before we go into detail on the description of the model as developed in the subsequent sections of this paper, we discuss some features of the traditional model of a market system and a general equilibrium in such an economy.

The important paper of Arrow and Debreu (1954) on the existence of a general equilibrium on a market economy with price taking agents already mentions two fundamentally different points of view to study this model. They argue that the model can be focused from a normative as well as a positive point of view. In this paper we focus on this model from a positive point of view. However, if we approach the Arrow–Debreu model positively, then we encounter some severe flaws. Next we will discuss some of these flaws to which our model as developed in this paper can be regarded as an extension of the traditional setting.

Firstly, we note that the concept of perfectly competitive markets as used in the Arrow–Debreu model is far from realistic. The extensive literature on other market forms, such as oligopoly and monopoly, confirms that this viewpoint is widely accepted in economic theory. Although in reality there is a mixture of many forms of competition, we observe that leadership and dominance is one of the major features on markets. In most cases producers and suppliers are dominant over consumers in the sense that they set prices or quantities, and consumers just act as followers. This specific form of dominance is also reflected in the models describing so-called "von Stackelberg" behaviour. In those models it is assumed that there is a (market) leader and some followers. Other forms of dominance are reflected in models of collusion and cartels versus fringes. From the literature, which studies these cases in the setting of perfectly competitive economies, we mention Shitovitz (1973) and Aumann (1973). The discussion above leads to the conclusion that on a market it is realistic to assume that there is some form of leadership, dominance, or inequality instead of equality among agents and pure price taking behaviour.

Secondly, the general equilibrium model of Arrow and Debreu reflects a situation of complete communication and completely free trade among the agents on the markets. In Debreu (1959) it is assumed that all market participants are able to communicate and trade freely with each other. We note that this again is too ideal a concept of a market. In reality it is obvious that there is only limited communication possible. If we accept that there are constraints in possibilities of trade among agents on the market, we get an asymmetric notion of a trader. It also implies that we do not longer have just an unstructured market, but some structured market. The asymmetric agents, acting on these structured markets, do not necessarily have to differ in individual characteristics, but certainly in their abilities to trade. We describe these incomplete trade structures by means of a deterministic graph, in which the nodes represent the agents and the edges represent (potential) trade relations. This is also the point of departure in the seminal work of Kalai, Postlewaite and Roberts (1978) on the three-person case. In that paper it also becomes clear that incomplete abilities to trade or communicate is in some respects complementary to our first remark on traditional general equilibrium models, namely the occurence of some form of dominance among agents on the market. The dominance then is a consequence of certain abilities to trade and/or communicate with other agents in the market.

Our third remark with respect to the Arrow–Debreu framework concerns the fact that in reality agents do not have complete information on all activities with respect to the supply and demand of a certain commmodity, let alone the individual attributes or characteristics of another agent in the market. The last point is extensively studied in the literature on incentive compatibility and the design of allocation mechanisms. (For some recent developments in this field we refer to Hammond (1987 and 1989).) In this paper we do not explicitly focus on the incompleteness of information concerning the prices as quoted on the markets in the economy, but remark that implicitly this is a natural consequence of limited abilities to communicate. In such a context it is clear that an agent does not have to have complete knowledge of all prices as quoted on the socially structured markets in the economy.

The points as discussed above make clear that the Arrow–Debreu model can not be approached as a proper description of reality. However, it is our opinion that the Arrow–Debreu model can be approached satisfactory as an idealization of reality, and therefore has essentially to be regarded from a normative point of view. The purely competitive Walrasian equilibrium concept is therefore used as a benchmark to test the outcomes of our model.

It is our point of view that the three remarks with respect to the traditional Arrow–Debreu model are closely related. In this paper we defend the argument that hierarchical positions in trade processes on the market as well as certain other asymmetries between the traders in the economy are resulting from natural constraints in the possibilities to trade. The lack of trade possibilities immediately implies the existence of asymmetric positions of agents. As a natural consequence we arrive at the conclusion that imperfect trade possibilities result in organization structures to overcome these imperfections and asymmetries. The presence of these trade organizations however result in positional advantages of certain traders over others, and thus to positional dominance and "von Stackelberg"-like environments. This reasoning leads us to the description of a market as a hierarchy consisting of price setting leaders and price taking followers.

On the other hand imperfections in possibilities to communicate immediately lead to asymmetries in knowledge of agents with respect to relevant aspects of market behaviour such as certain prices, which are quoted on the market. This has important consequences with respect to the abilities to form a coalition and the establishment of contracts between traders on the market. The consequences with respect to coalition formation are studied in Gilles and Ruys (1989). From this we may conclude that behaviour, which is based on Edgeworth's recontracting principle, is highly sensitive with respect to the communicative abilities of the agents in the market.

The *principal feature* of our model is therefore the lack of complete possibilities to trade as well as to communicate on the markets in the economy. This implies that we take both features as described above into full account, namely hierarchical trade positions on the market due to imperfect trade possibilities and constraints in coalition formation due to constraints in communication. Both features are therefore represented in our model of a trade economy. The model thus consists of a description of a hierarchical market structure in which agents act non-cooperatively and a configuration describing the coalitions, which are allowed to act in the cooperative trade processes based on Edgeworth's recontracting principle. This implies that in our model we describe a market in which there is *non-cooperative* as well as *cooperative* trading behaviour. Our point of departure is that the trade structures, representing each of these behaviouristic spheres, have to be separated.[1]

[1] Our main argument is that (non-cooperative) hierarchical trade relations are essentially different, and therefore to be separated from the trade relations to be used in cooperative behaviour. We view

The model therefore consists of two separated trade structures. The structure, which describes the non-cooperative trade relations, is built on the notion of a *network* as developed by Gilles and Ruys (1988) and Gilles, Ruys and Shou Jilin (1989) in the setting of an economy with an arbitrary communication structure. Networks can be interpreted as collections of agents with positional advantages over other agents, i.e., agents outside the network, in the market. The natural description of a hierarchical trade organization turns out to be a nested sequence of networks within networks. We assume that members of the higher echelons in the hierarchy act as price setting leaders, while the members of the lower echelons in the hierarchy act as price taking followers. This describes the well known non-cooperative "von Stackelberg" leader–follower behaviour in the setting of a hierarchically structured market.

The second component of our model of a trade economy is the description of the configuration, which serves as the basis for the cooperative trading processes on the market according to the Edgeworthian recontracting principle. Given the prevailing hierarchy on the market this *re-trade configuration* is given as a mapping which assigns to every agent on the market a collection of traders of which he or she has enough information to cooperate with in the Edgeworthian sense. This implies that for every trader in the assigned collection the agent knows the quoted prices within the hierarchy, which are relevant for that particular trader. If this knowledge is mutual, then it is clear that these agents are able to form a coalition and to trade if that is profitable for both for them. This rule of mutual information gives us the collection of truly allowable coalitions within the cooperative trading processes on the market.[2]

Within the setting of a trade economy as described above we establish a *two stage model of trade*. In the first stage any trader chooses a unique predecessor in the relevant network within the hierarchy. It is assumed that this predecessor acts as the only trading partner in the higher network within the hierarchical trade structure on the market. The second stage of the model is that of actual trade. Given the non-cooperative hierarchical trade structure, that results from the first stage of the

the hierarchically structured market as the "official" market, while Edgeworthian recontracting is viewed as a "gray" market activity resulting as a reaction to imperfections in the "official" market.

[2] We note that in these cooperative processes there is no explicit pricing. It is therefore a description of re-trading or arbitrage processes rather than the primary trading processes on the market. These primary trading processes are recognized to be non-cooperative, and are assumed to take place within the setting of the hierarchical trade organization.

model, any agent determines his or her optimal strategies given his or her information and trading possibilities.

It is assumed that in the non-cooperative trade processes an agent acts as a price setter for those traders, who choose him or her as their unique predecessor in the first stage of the model, while he or she acts as a price taker with respect to his or her chosen predecessor in the hierarchical trade structure on the market. The optimal non-cooperative actions of the agent are determined with respect to the reaction functions of the successors[3] and the cooperative trade processes in the economy. It is clear that this describes the "normal" von Stackelberg behaviour within a hierarchically structured situation.

Truly allowable coalitions are assumed to take part in the cooperative trade processes if that is profitable for all members of these coalitions. This implies that besides the normal "blocking" behaviour á la Edgeworth we can distinguish cooperative trade if there is any price discrimination between members of the same allowable coalition. In those cases it is possible for such a coalition to generate infinite profits by buying and selling at the same time. These arbitraging trade processes lead to the elimination of all price discrimination with respect to members of the same allowable coalition.

An equilibrium in the second stage of our two stage model of trade is defined as a price–allocation system, which is individually strategically optimal, given the possibilities of the traders in the hierarchy, as well as cooperatively stable in the sense that no allowable coalition is able to improve upon that price–allocation system. With the use of this equilibrium in the second stage of our model, we can construct an equilibrium in the first stage of our model. Every agent is chosing a predecessor in the hierarchy such that, given the choices of the other agents and the belonging Edgeworthian re-trade configuration, his or her resulting end bundle in the equilibrium of the second stage is optimal, i.e., maximal with respect to the possibilities available in the given hierarchy and re-trade configuration.

The equilibrium concept as described above is based on the assumption that cooperative *as well as* cooperative behaviour are essential features in the trade processes that lead to a sustainable stable allocation. Non-cooperative behaviour only is not sufficient to guarantee the achievement of such a sustainable outcome, since too little

[3]The successors of a certain trader are those agents, who have chosen that particular trader as their predecessor in the first stage of our model of trade.

a number of trade relations is used in these trading processes. On the other hand cooperative behaviour is far too weak to be sufficient to sustain such an allocation within the setting of an economy with limited communication.

This remark is confirmed in our analysis of Walrasian allocations in the setting of our two stage model of trade in a hierarchically structured market. The main result of this analysis states that in re-trade configurations, which chain all agents in the market through some sequence of feasible coalitions, the equilibrium in the second stage of our model is indeed Walrasian. This theorem implies that the combination of a hierarchical market structure and a rich enough re-trade configuration guarantees, even in a finite market, the generation of a Walrasian allocation. Hence, this learns that besides largeness of the market and the implementation of strategic behaviour, the organization of the market is of major importance with respect to the analysis of perfect competition.

The paper is organized as follows. In Section 2 we discuss the problem of describing a hierarchical trade organization as a natural pattern in a market with imperfections in trade relations. Section 3 presents the formal development of the two stage model of trade and the belonging equilibrium concept. In Section 4 we specifically focus on two kinds of Edgeworthian re-trade configurations and the consequences of their implementation in the equilibrium of the model as developed in Section 3. We conclude this paper with some remarks concerning the possible extensions of the model and the problem with respect to the proof of the existence of an equilibrium in our two stage model of trade.

2 Hierarchical Trade Structures

The first step in the construction of a model of a trade economy with an incomplete communication or trade structure is the analysis of the consequences of the limitation of trade possibilities with respect to positional asymmetries. As argued in the introduction the incompleteness of the trade structure leads in a natural way to positional asymmetries and dominance. It is our purpose to show that an incomplete communication structure naturally leads to *hierarchical trade organizations*, of which the existence is justified by the purpose of trade itself. It is therefore our point of departure that such a hierarchical trade organization exists just because of the need to overcome imperfections in the trade possibilities of the agents in the economy.

Formally we depart in our analysis from a population, which describes agents with their potential trade relations with other agents in the economy.

Definition 2.1 *The pair (A, R) is a* **population** *if A is a finite set of economic agents and $R \subset \{ \{a, b\} \mid a, b \in A\}$ is a collection of undirected relations such that for every $a \in A$ it holds that $\{a, a\} \in R$ and for every two agents $a, b \in A$ there exists a finite sequence of agents $c_1, \ldots, c_n \in A$ such that $c_1 = a$, $c_n = b$, and for every $j \in \{1, \ldots, n-1\}$ it holds that $\{c_j, c_{j+1}\} \in R$.*

If two agents $a, b \in A$ are directly related in R, i.e., if $\{a, b\} \in R$, then we can interpret it as that these agents are able to trade with each other. The choice whether this trade possibility is actually used will be depending on the behaviour and global position of both traders in the trade structure of the economy. A pair (A, R), which satisfies the definition above, therefore represents the potential trade or communication structure in the economy.[4]

The first step in our analysis is to introduce certain groups or coalitions of traders, which have a positional advantage in the trade structure. This positional advantage consists of the ability of that group — as a whole — to regulate all trade flows in the economy. We note that, given the population, there may exist several of these groups in the economy.

To formalize this notion of positional advantage, we introduce some additional notation. Let $E \subset A$ be some group of traders, then $(E, R|E)$ describes the trade possibilities within this group, i.e., $R|E = \{ \{a, b\} \in R \mid a, b \in E\}$. Finally we denote by $\mathcal{P}(A) = 2^A \setminus \{\emptyset\}$ the collection of all nonempty subsets of the collection of traders A.

Definition 2.2 *Let (A, R) be a population of traders, and let $E \in \mathcal{P}(\mathcal{A})$.*

(a) *A collection of traders $N \subset E$ is a* **network** *in $(E, R|E)$ if it satisfies the following properties:*

Reachability

For every agent $a \in E$ there exists an agent $b \in N$ such that $\{a, b\} \in R$.

Connectivity

The graph $(N, R|N)$ is connected.

[4] In mathematical terms the pair (A, R) is a graph, which is undirected and connected.

Minimality

There is no agent $a \in N$ such that the subgroup $N \setminus \{a\}$ satisfies reachability as well as connectivity.

(b) *The mapping $\Psi: \mathcal{P}(A) \to 2^{\mathcal{P}(A)}$ is the* **network mapping** *of (A, R) if for every non-empty collection of traders $E \in \mathcal{P}(A)$ it holds that*

$$\Psi(E) = \{N \subset E \mid N \text{ is a network in } E\}.$$

In order to interpret the definition above we restrict ourselves to the analysis of a network in A, i.e., choose $N \in \Psi(A)$. It is clear that such a network, as defined above, is just a description of a group of traders, which *in its totality* has a positional advantage in the trade structure of the economy. We therefore interpret a network in $E \in \mathcal{P}(A)$ as a potential organization, which is able to regulate all trade processes within E. This is recognized by investigating the three basic properties of a network. Firstly, a network is directly reachable by any agent in E. Secondly, it is (potentially) able to provide any other agent in E with some bundle of commodities. The third property is a crude efficiency criterion, which requires that no agent in the network is superfluous in the sense that his or her removal affects the reachability or connectivity properties of that group. Thus, any agent in a network is essential in maintaining one of these properties.

It may be clear that a network in a certain group of traders $E \subset A$ as defined above is a coalition of traders with an important positional advantage over the other agents in that group. It is also clear that in certain cases there may not even exist such a network. On the other hand the collection of networks $\Psi(E)$ may be quite large. The next lemma gives some insight into the question after the existence of networks. The proof of the lemma is trivial.

Lemma 2.3 *Let Ψ be the network mapping of the population (A, R). Then for a non-empty collection of traders $E \in \mathcal{P}(A)$ it holds that $\Psi(E) \neq \emptyset$ if and only if $(E, R|E)$ is connected.*

A corollary of the lemma above is that for a population (A, R) it holds that $\Psi(A) \neq \emptyset$, since (A, R) is assumed to be connected. We moreover note that this lemma shows that the definition of the network mapping is consistent. Namely, for every collection of traders $E \in \mathcal{P}(A)$ and for every network $N \in \Psi(E)$ the sub-graph $(N, R|N)$ is indeed connected, and thus $\Psi(N) \neq \emptyset$.

We do not consider a network to be a proper description of a trade organization within the setting of a trade economy as described previously. We therefore introduce the notion of a hierarchical system of networks, or simply a *hierarchy*, as a nested sequence of networks within networks. The economic foundation of this notion is that certain trade flows may be handled by some network, but that *within* such a network the handling of these flows has also to be described properly. This involves the consideration of networks within that network handling these trade flows in the economy. In that sense we arrive at a hierarchically ordered sequence of networks within networks, all of which are potentially able to handle the trade flows of commodities in the economy.

This short verbal introduction of the notion of hierarchy consequentially recognizes that incomplete abilities to trade naturally lead to positional asymmetries and organization structures in the economy to overcome these defects in the trade structure. The notion of hierarchy seems a natural expression of such an organization structure, which is necessary in the presence of imperfections in the trade infra-structure of the economy.

Definition 2.4 *A finite sequence of non-empty collections of traders $N_1, \dots, N_k \in \mathcal{P}(A)$ is a* hierarchy *in the population (A, R) if it satisfies the following properties:*

1. $N_1 = A$;

2. *for every number $j \in \{1, \dots, k-1\}$ it holds that*

$$N_{j+1} \in \Psi(N_j) \neq \{N_j\} \quad ;$$

3. $\Psi(N_k) = \{N_k\}$.

The collection of all hierarchies in (A, R) is denoted by $\mathcal{H}$.

We view a hierarchy as a finite sequence of "network levels", in which each level takes care of the regulation of all trade flows which occur on the previous level. The first or lowest level is obviously the total economy, and the second level is a "normal" network within the economy. This network regulates all trade flows within the whole economy, i.e., the previous level. The third level is a network within the network on the second level. Obviously, this "super" network takes care of the regulation of all trade flows within the "normal" network, and, implicitly, of many trade flows within the total economy.

The next result shows us that the definition of a hierarchy as given above is consistent. Secondly, we show that a hierarchy naturally ends within a singleton, i.e., the highest level of the hierarchy is a singleton.

Lemma 2.5 *Let (A, R) be a population.*

(a) *There exist hierarchies in (A, R), i.e., $\mathcal{H} \neq \emptyset$.*

(b) *For every hierarchy $\xi = (N_1, \ldots, N_k) \in \mathcal{H}$ there exists an agent $a \in A$ such that $N_k = \{a\}$.*

The proof of the assertions of the lemma above can be found in Gilles (1990). The main property is that hierarchies have a natural length, namely it stops as $\Psi(N_k) = \{N_k\}$.[5] In the sequel we denoted the length of a hierarchy $\xi \in \mathcal{H}$ by $\ell(\xi)$.

We remind that a hierarchy is a description of an organization structure, which is potentially able to regulate all trade flows in the economy. As discussed above this organization structure consists of a number of levels. Each level is represented by some network within the previous level. It is made clear that such a level regulates all trade flows on the previous level, except those which the previous level itself can regulate. This interpretation is supported by redefinition of the system of levels into a system of *echelons*.

We take a fixed hierarchy $\xi \in \mathcal{H}$ with length $\ell(\xi) = k$. Denote this hierarchy by $\xi = (N_1, \ldots, N_k)$, where $N_1 = A$. Now, we can rewrite the hierarchy ξ as a finite sequence $\tilde{\xi} = (S_1, \ldots, S_k)$, which is defined as follows:

- For every $1 \leq j \leq k - 1$ we define $S_j = N_j \setminus N_{j+1}$.
- Finally, we take $S_k = N_k$.

We immediately may conclude that $\tilde{\xi}$ is a partition of A, i.e., for every $i, j \in \{1, \ldots, k\}$, with $i \neq j$, it holds that $S_i \cap S_j = \emptyset$, and $\cup_{i=1}^{k} S_i = A$. The sequence $\tilde{\xi}$ is denoted as the *echelon partition* induced by the hierarchy ξ, and its elements $S_1, \ldots, S_k$ are called the *echelons* belonging to the hierarchy ξ.

[5] We remark that this occurs only when the set is a singleton. Hence, we conclude that N_k is a singleton, whom we denote as the "topman" of the hierarchy.

Economically the "higher" echelons in the echelon structure are numbered by a higher integer in the (finite) sequence $1, \dots, k$, where $k = \ell(\xi)$. Thus we have generated a natural subdivision of the population into echelons with different positions as imposed by the hierarchy ξ on that population in the trade economy. To return to the discussion of the hierarchy as an organization structure in the economy that takes care of trade regulation, we note the following. An echelon, say S_i, takes care of all trade flows between members in the lower echelon S_{i-1}. This is done either directly by acting as a direct intermediary trader or indirectly by shifting the regulation to the higher network N_{i+1}. There the trade flow is directed towards another agent in the echelon S_i, who can take care of the provision of an agent in the lower echelon S_{i-1} to which the trade flow has to be directed ultimately.

The next definition is describing the structure as imposed on the economy in case the hierarchy ξ is regulating all trade flows in the economy. It consists of all trade relations, which are *potentially relevant* for the hierarchy to regulate the trade flows in the economy.

Definition 2.6 *Let $\xi \in \mathcal{H}$ and let $\tilde{\xi}$ be the induced echelon partition by ξ. The pair (A, R_ξ) is the* **echelon structure** *induced by ξ if (A, R_ξ) is a subgraph of the population (A, R) such that $R_\xi = \bigcup_{j=1}^{k-1} W_j \cup \{ \{a,a\} \mid a \in S_k\}$, where for every $j = 1, \dots, k-1$*

$$W_j = \{ \{a,b\} \mid a \in S_j \text{ and } b \in N_{j+1}\} \cup \{(a,a) \mid a \in S_j\}.$$

As a technical remark we mention that for every hierarchy $\xi \in \mathcal{H}$, the echelon structure (A, R_ξ) is a connected sub-graph of the population (A, R). Further technical properties of an echelon structure can be found in Gilles (1990).

In a trade economy as described in this section all agents in a particular echelon are having approximately the same position in the hierarchical structure of the economy.[6] In that case, the trade relations as described in the echelon structure are representing the activated trade relations in the presence of that particular hierarchy. So an echelon structure is describing possible trade patterns in the economy, and obviously can be viewed as a part of a more stage process. In the first stage a social contract is made: A hierarchy is "chosen" as the organisation structure in this exchange economy. Every

[6]Provided that this particular hierarchy is regulating the ocurring trade flows, all agents in a certain echelon are treated in the same way with respect to the regulation of their trade.

agent in the economy conforms his individual behaviour with respect to this contract. Now if, in the second stage, actual trade takes place, it only can go through the channels as described in the echelon structure as induced by the contracted hierarchy.

The next step in our analysis is to investigate the choices, which a single trader can make in case of actual trade. Within the echelon structure all agents, except the "topman" in the highest echelon, are able to choose from a number of intermediary agents in the higher network to actually perform trade in the next stage of the trade process. Thus, the trader chooses *one* intermediary agent in the higher echelon, before engaging into actual trade.

In the next definition we introduce some notational conventions with respect to the choice processes, which occur in case of the realisation of trade opportunities. The analysis only regards the choices of an intermediary agent in order to be able to trade, and *not* the trade process and contracting itself.

Definition 2.7 *Let $\xi = (N_1, \ldots, N_k) \in \mathcal{H}$ be a hierarchy, and let $\tilde{\xi} = (S_1, \ldots, S_k)$ be the echelon partition induced by ξ.*
The **echelon relation mapping** *is a correspondence $P_\xi : A \to 2^R$, which for every $1 \leq j \leq k-1$ and every $a \in S_j$ is given by*

$$P_\xi(a) := \{\ \{a, b\} \in R \mid b \in N_{j+1}\} \subset R_\xi \quad ;$$

and for $a \in S_k$ is given by $P_\xi(a) := \emptyset$.

The echelon relation mapping is a description of the potentially relevant trade relations with a network in case a certain hierarchy occurs. The next step in our modelling procedure is to assume that in case trade takes place an agents trades at most with one representative of the network he or she has to deal with. This is equivalent to the assumption that an agent $a \in A$ activates only one potential trade relation in the collection $P_\xi(a)$. The resulting structure of activated trade relations can be described as follows.

Definition 2.8 *Let $\xi = (N_1, \ldots, N_k) \in \mathcal{H}$ be a hierarchy, and let $\tilde{\xi} = (S_1, \ldots, S_k)$ be the echelon partition induced by ξ.*
An **echelon tree** *induced by ξ is a directed subgraph (A, W) of the echelon structure (A, R_ξ) such that there exists a function $t : A \setminus S_k \to A \setminus S_1$, which satisfies the properties that*

$$W = \{(a, t(a)) \mid a \in A \setminus S_k\} \subset A \times A$$

and for every agent $a \in A \setminus S_k$ *it holds that* $\{a, t(a)\} \in P_\xi(a)$.
The collection of all echelon trees induced by ξ *is denoted by* $\mathcal{T}_\xi$.

It is clear that to every echelon tree $(A, W) \in \mathcal{T}_\xi$ there belongs a unique function t as described in the definition. In the sequel we will denote this function t as the *predecessor function* of (A, W). As a natural consequence we denote for every agent $a \in A \setminus S_k$ the agent $t(a) \in A \setminus S_1$ as the *predecessor* of a in (A, W). The next lemma gives an alternative description of the notion of echelon tree.

Lemma 2.9 *Let* $\xi \in \mathcal{H}$ *be a hierarchy in the population* (A, R). *The pair* $(A, W) \in \mathcal{T}_\xi$ *is an echelon tree if and only if* (A, W) *is a directed subgraph of the echelon structure* (A, R_ξ) *such that*

(i) (A, W) *is weakly connected ;*

(ii) *for every agent* $a \in A$: $\#[P_\xi(a) \cap \overline{W}] \leq 1$, *where* $\overline{W} = \{\ \{a, b\} \mid (a, b) \in W\}$, *and*

(iii) *for every* $1 \leq j \leq k-1$ *and every agent* $a \in S_j$ *there exists an* $i \in \{j+1, \dots, k\}$ *such that*

$$(a, b) \in W \quad \textit{implies} \quad b \in N_i.$$

The idea behind our model of a trade economy is two-sided. Firstly, in the pre-trade stage a certain hierarchy is chosen, and with it the echelon structure is determined.[7] Secondly, within the chosen hierarchical structure actual trade will take place. Therefore the main structural concepts in the regulation of trade are echelon trees. In many decision instances we suppose that an agent chooses *one* predecessor in the echelon structure to trade with. This causes the emergence of an echelon tree within the echelon structure. In the next section we discuss a model which is build according to these principles.

[7]It is supposed that the choice of a hierarchy is done according to some *social contract*. The precise determination which hierarchies are fitted to serve as trade organization structures will be subject of future research. The preliminaries of this study of such a social contract is taken up in van den Brink (1989).

The discussion in the previous paragraph condenses into the consensus that it is appropriate to assume that realizations of trade flows occur within echelon trees instead of in the more general setting of echelon structures. We therefore conclude this section with a final definition, which gives some additional notation to handle this basic assumption in our modelling of trade processes. It also gives a foundation to the equilibrium as introduced in this paper. It gives a full description of the notion of predecessor, and the usefulness of this notion with respect to the description of an echelon tree.

This is confirmed in the next proposition, which lists some properties of an echelon tree. These properties are immediate consequences of the definitions as given earlier.

Proposition 2.10 *Let $\xi \in \mathcal{H}$ be a hierarchy in (A, R), and let $(A, W) \in \mathcal{T}_\xi$ be an echelon tree induced by ξ. Define $\overline{W} := \{ \{a, b\} \mid (a, b) \in W \} \subset R$, then the following properties hold:*

(a) *$(A, \overline{W})$ is a spanning tree of the echelon structure (A, R_ξ) and the population (A, R).*

(b) *For every $1 \leq j \leq k-1$ and every agent $a \in S_j$ it holds that*

$$\#[P_\xi(a) \cap \overline{W}] = 1.$$

(c) *For the unique topman $a \in S_k$ it holds that*

$$\#[P_\xi(a) \cap \overline{W}] = 0.$$

3 Hierarchically Structured Trade Economies

In this section we introduce and analyse a model of a pure trade economy, in which trade flows are explicitly regulated by a hierarchy in the trade structure of the economy. We suppose that the behaviour of the members of the hierarchy is essentially non-cooperative. This implies that the members of a higher echelon regulate trade between traders in the lower echelons by setting prices. We therefore implicitly deal with a Stackelberg-like equilibrium concept. In such an equilibrium concept it is assumed that network members in the hierarchy set prices for all binary trade flows in the economy, and thus are *leaders*. The traders in lower echelons are *followers* in the sense that they act as price takers with respect to the higher members of the hierarchy. We thus arrive at a *hierarchical Stackelberg equilibrium concept*, in which members of higher echelons are leaders with respect to members of the lower echelons.[8]

Besides the non-cooperative behaviour as described in our hierarchical Stackelberg equilibrium, we assume that the traders in the economy can act cooperatively if that is profitable. Given the prices and allocations as set in the hierarchical trade structure the traders in the economy are assumed to be able to recontract in the sense of Edgeworth's recontracting principle. This implies that given a price-trade-allocation as emerging from the hierarchically organized trade flows, there exist additional trade possibilities, which are used cooperatively by the traders. These additional (binary) trade relations are described *independently* of the hierarchical structure in the economy. This assumption gives us two trade structures, namely the hierarchical non-cooperative trade structure, in which there is regulation by pricing, and a cooperative Edgeworthian trade configuration, in which there is no explicit pricing. We note that recontracting only occurs if the non-cooperative hierarchical system is in disequilibrium, in the sense that traders can be better off in case of Edgeworthian recontracting or re-trading. Price-allocation systems, which are optimal in the sense that no trader can improve by re-trading à la Edgeworth, are called *re-trade proof* price-allocation systems. Thus, cooperative behaviour is initiated by imperfections in the hierarchically structured market.

Next we formally define a trade economy as a combined system of a hierarchical trade structure and a collection of binary trade relations describing the possibilities to re-trade in the Edgeworthian sense.

Definition 3.1 *A* **trade economy** *is a tuple* $\mathbf{E} = ((A, \xi, R_\xi), \Im, g)$*, where the following properties hold.*

- *The pair* (A, R_ξ) *is a population.*
- ξ *is a hierarchy in the population* (A, R_ξ)*, such that the pair* (A, R_ξ) *is exactly the echelon structure belonging to* ξ*.*

[8]For a comparable recent study we refer to Robson (1988). There a hierarchical Stackelberg equilibrium is analysed in the setting of a model with entry, i.e., it studies hierarchical entry in an oligopolistic market.

- *The mapping* $\Im: \mathcal{T}_\xi \times A \to 2^A$ *assigns to every agent* $a \in A$*, in case some echelon tree* $(A, W) \in \mathcal{T}_\xi$ *occurs , the potential coalition partners* $\Im_W(a) := \Im((A, W), a) \subset A$*, which describes the trade relations of agent* $a \in A$ *to be used for Edgeworthian re-trading.*

- *The mapping* $g: A \to \mathcal{P}_{mo} \times \mathbb{R}^\ell_+$ *is a characterization, which assigns to every agent* $a \in A$ *a monotonic*[9] *and continuous preference relation* $\succ_a \in \mathcal{P}_{mo}$ *and an endowment* $w_a \in \mathbb{R}^\ell_+$*, where* ℓ *is the number of commodities in the economy.*

- *For every agent* $a \in A$ *we denote* $g(a) = (\succ_a, w_a) \in \mathcal{P}_{mo} \times \mathbb{R}^\ell_+$*, and we suppose that*

$$w := \sum_{a \in A} w_a \gg 0.$$

It is evident that in Definition 3.1 a trade economy is defined by three main items. Firstly, the triple (A, ξ, R_ξ) describes the hierarchical non-cooperative trade structure in the economy. Secondly, the mapping $\Im$ describes the trade structure, which can be used for Edgeworthian re-trading. We note that this *re-trade configuration* is defined independently of the echelon structure (A, R_ξ). Finally, the mapping g is a characterization of the agents in the population. It assigns to every agent in the economy an endowment and some preference relation on the commodity space as his or her individualized attributes.

With respect to the economy as defined above we make some important assumptions on the behaviour of the agents. Firstly, the actions in the hierarchical non-cooperative trade structure and those in the re-trade processes in the configuration as described by the mapping $\Im$ are essentially independent, in the sense that agents are able to act freely in both modes of behaviour in the trade economy, given the trade relations in both configurations. Rationality implies that agents coordinate these actions in the non-cooperative trade structure respectively the cooperative re-trading processes. Secondly, we assume that all agents in the economy are rational and that it is common knowledge that they are rational. Thirdly, all agents in the economy know the hierarchical trade structure of the economy *globally*. We are now able to describe the two stages of our model of trade as follows.

[9]A preference relation $\succ$ is monotonic if for every $x, y \in \mathbb{R}^\ell_+$ it holds that if $x \geq y$, and $x \neq y$, then $x \succ y$.

STAGE I : CHOICE OF INDIVIDUAL TRADE PARTNERS

Let E be a trade economy as defined in Definition 3.1. Moreover, let $k = \ell(\xi)$ be the length of the hierarchy ξ. Now the echelon structure (A, R_ξ) describes all trade relations in the non-cooperative hierarchical trade structure of the economy. Thus the set $P_\xi(a)$ denotes to agent $a \in A$ a collection of potential trade relations with the higher echelons in this structure. We assume that in Stage I of the model an agent $a \in A \setminus S_k$ chooses *one and only one* potential trade relation $\tau_a \in P_\xi(a)$ to be activated in the second stage of the model, namely the stage of actual trade. This implies that an agent chooses exactly one potential trade relation to be activated in the hierarchical trade structure. For any agent $a \in A \setminus S_k$ let $\{a, b\} \in R_\xi$ denote the chosen trade relation to be activated. Then we know that $b \in N_{j+1}$, when $a \in S_j$ $(1 \leq j \leq k-1)$. Now we call the agent $b \in N_{j+1}$ the *predecessor* of $a \in S_j$.[10] The function $t: A \setminus S_k \to A \setminus S_1$ assigns to every agent $a \in A \setminus S_k$ his or her chosen predecessor $t(a) \in A \setminus S_1$. The function t defines an echelon tree $(A, W) \in \mathcal{T}_\xi$ with $W = \{\tau_a \mid a \in A \setminus S_k\}$, where $\tau_a = (a, t(a))$. Finally we denote for every agent $a \in A \setminus S_1$ the inverse of t by

$$t^{-1}(a) := \{b \in A \mid t(b) = a\}.$$

STAGE II : ACTUAL TRADE

Let (A, W) be the echelon tree that results from the choice of predecessors as described in Stage I of the model. This echelon tree describes the trade structure with respect to non-cooperative trading behaviour. Moreover, the mapping $\Im_W$ describes the trade structure with respect to the Edgeworthian re-trading behaviour in the economy. We discuss these two configurations subsequently as follows:

There is a leader – follower behaviour within the trade structure as described by the echelon tree (A, W). It is supposed that predecessors act as price setters with respect to their successors. Formally this implies that for an agent $a \in A \setminus S_k$ his or her predecessor $t(a)$ sets prices on the activated trade relation $\tau_a = (a, t(a)) \in R_\xi$.

[10] Thus in Stage I of our model any agent in the population, except the "topman", is choosing a predecessor. This predecessor acts as the unique regulating agent for all trade of agent a with the higher echelons.

Any agent $a \in A$ is able to trade with agents in the collection $\Im_W(a) \subset A$ in the sense of Edgeworth's recontracting principle. Given the price-allocation system as emerging by behaviour of the agents in the non-cooperative trading processes within the setting of the echelon tree (A, W), agents are using these possibilities as described by the set $\Im_W(a)$ when this is profitable. This implies that if agents are able to reach a better end bundle by re-trading on some existing price-allocation system, this possibility will be used. These Edgeworthian re-trade processes can also be regarded as outcomes of an arbitrage system. This is also sustained by our assumption that any equilibrium price-allocation system should be stable with respect to such re-trading processes, i.e., all re-trading possibilities are not profitable.

In the sequel we discuss the two stages as globally described above. Evidently this will be done in reverse order, i.e., first we discuss the behaviour and equilibrium in the second stage of our model of trade before we turn to a description of equilibrium in the first stage of the model. First we state some assumptions.

3.1 Some assumptions

Let $\mathbf{E} = ((A, \xi, R_\xi), \Im, g)$ be a trade economy as defined in Definition 3.1. We make the following assumptions with respect to the different items in the trade economy $\mathbf{E}$.

1. For every agent $a \in A$ we assume that the preference relation $\succ_a \in \mathcal{P}_{mo}$ can be represented by a strictly quasi-concave and continuous utility function on the commodity space $\mathbf{R}^\ell_+$.
   Moreover we assume that the agents always choose a unique action, given the environment in which they operate. So, given the prices set by the higher echelon they choose a maximal end bundle in their budget set, given the re-trading activities and the prices set by the other agents in the economy. If this set is convex, then obviously the bundle is unique, but in case of non-convexities it is still assumed that the agents choose one of the optimal actions according to some given rule. Essentially it is assumed that other agents can describe the reaction curve of any other agent in the economy. We can therefore speak of

demand functions and *reaction functions* instead of correspondences.[11]

2. In the hierarchical echelon tree (A, W), which results from a choice procedure in Stage I of the model, we make the following assumptions:

   - An agent $a \in A \setminus S_1$ acts as a price setter on the trade relations $\{\tau_b \mid b \in t^{-1}(a)\} \subset W$.
   - An agent $a \in A \setminus S_k$ acts as a price taker with respect to the activated trade relation $\tau_a \in W$.

   It is clear that these two assumptions give a complete account of the non-cooperative trading behaviour of the agents in the setting of a hierarchical trade structure. Agents in the higher echelons evidently act as "leaders", while agents in the lower echelons act as "followers". This gives a sufficient description of the hierarchical "von Stackelberg" equilibrium as mentioned before.

3. The mapping $\Im\colon \mathcal{T}_\xi \times A \to 2^A$ describes the re-trading possibilities of the agents in the setting of the emergence of one echelon tree in the trade economy. Let (A, W) be such an echelon tree. Then we assume the following:

   - For every agent $a \in S_1$ in the lowest echelon it holds that
     $$a \in \Im_W(a).$$
   - For every other agent $a \in A \setminus S_1$ it holds that
     $$\{a\} \cup t^{-1}(a) \subset \Im_W(a).$$

   The assumptions as stated above give the minimal requirements with respect to re-trade possibilities in the setting of an occurring echelon tree.

4. In combination of the previous two assumptions on respectively the non-cooperative hierarchical trade structure and the cooperative re-trade configuration, we assume that for every emerging echelon tree $(A, W) \in \mathcal{T}_\xi$, every agent $a \in A$ knows the price as set on the relation $\tau_b = (b, t(b)) \in W$ for every related agent $b \in \Im_W(a)$.

[11] We remark that this assumption is indeed very strong. It will be one of the major difficulties in the proof of the existence of an equilibrium to design individual attributes which generate such nice functions instead of correspondences. The main difficulty lies in the fact that the budget sets have to be convex to arrive at a unique optimal choice of actions. This issue will be addressed in a subsequent paper.

The assumptions as discussed above are to be used in the construction and definition of an equilibrium in our two stage model of a trade economy. This will be done in the next two subsections, which define an equilibrium concept for subsequently the two stages of our model of trade.

3.2 Actual trade

In this subsection we concentrate on Stage II of our model of a trade economy with a hierarchical structure and an Edgeworthian arbitrage system. We suppose that in the trade economy $\mathbf{E} = ((A, \xi, R_\xi), \Im, g)$ there has been resulting an echelon tree $(A, W) \in \mathcal{T}_\xi$ from Stage I of the model. This echelon tree is completely described by the predecessor function $t: A \setminus S_k \to A \setminus S_1$. Finally, we accept the assumptions as made in the previous subsection. The next definition introduces the main tool of our analysis. The non-cooperative behaviour in the hierarchical trade structure results into a pattern of net trades on the activated trade relations, and some vector of prices on each of these binary trade relations. This combination is described by a *trade-price-system*, which also implies the existence of some *allocation* of commodities over the agents in the population.

Definition 3.2 *A* **trade–price–system** *in the economy* $\mathbf{E}$ *is a pair* $(\hat{f}, q)$, *where*

- $\hat{f}: W \to \mathbb{R}^\ell$ *is a* **net trade function** *denoting to every trade relation* $\tau_a = (a, t(a)) \in W$, *where* $a \in A \setminus S_k$, *a bundle of net trades from* $t(a)$ *to* a *such that*

$$\sum_{b \in t^{-1}(a)} \hat{f}(b, a) \leq \hat{f}(a, t(a)) + w_a.$$

- $q: W \to S^\ell := \{x \in \mathbb{R}^\ell_+ \mid \sum x_i = 1\}$ *is a* **price system** *denoting to every binary trade relation* $\tau_a = (a, t(a)) \in W$, *where* $a \in A \setminus S_k$, *some price vector* $q(\tau_a) \in S^\ell$ *at which the trade as described by the net trade function* $\hat{f}$ *takes place.*

From the definition above it is easily concluded that any trade–price–system $(\hat{f}, q)$ involves the existence of a net trade function, a price system, and an allocation. The net trade function denotes to every trade relation in the official structure of the economy, resulting from Stage I of the model, some bundle of net trades between the predecessor and the successor on that (activated) trade relation. As an easy consequence of the definition it follows that

$$\sum_{a \in A \setminus S_k} \hat{f}(a, t(a)) = \sum_{a \in A \setminus S_1} \sum_{b \in t^{-1}(a)} \hat{f}(b, a). \tag{1}$$

The price system denotes to every activated trade relation, and therefore to every trade within the economy, some price vector. Taking into account the leader–follower assumption, we know that the successor determines the size of the net trade bundle, while the predecessor determines the price-vector of the trades on that particular trade relation.

Resulting from a trade-price-system $(\hat{f}, q)$ is also an **allocation**, which is defined as a mapping $f: A \to \mathbf{R}^\ell$, denoting to every agent a bundle of total net trades, given by

$$f(a) = \begin{cases} \hat{f}(a, t(a)) - \sum_{b \in t^{-1}(a)} \hat{f}(b, a) & \text{for} \quad a \in A \setminus S_k \\ \sum_{b \in t^{-1}(a)} \hat{f}(b, a) & \text{for} \quad a \in S_k \end{cases}$$

Note that the allocation is only depending upon the net trade function $\hat{f}$ and *not* on the price system q. The two conditions on the net trade function also imply some similar, induced conditions on the allocation. Both can be formalized as follows:

$$\sum_{a \in A} f(a) = 0 \quad \text{and} \tag{2}$$

$$\text{for every agent } a \in A \setminus S_k \text{: } f(a) + w_a \geq 0 \tag{3}$$

The conditions on net trade functions and allocations can easily be interpreted as feasibility constraints. Firstly, the total bundle that is traded in the trade process is feasible in the normal sense, namely that the trades are in balance: $\sum (w_a + f(a)) \equiv w$, where $w_a + f(a)$ is the bundle of agent $a \in A$ after the trade-process.[12] Secondly, it is assumed that a single trader cannot be trading more than he or she owns, i.e., his or her total trade does not exceed his or her endowment.

Trade–price–systems obviously describe (potential) outcomes of non-cooperative trade processes in the hierarchical trade structure as described by the echelon tree (A, W). Now we turn to the description of the Edgeworthian arbitrage processes in the trade economy, which take place within the setting of the trade structure as described by the mapping $\Im_W: A \to 2^A$. The first step in the description is to determine which coalitions are allowed to form and come together to re-trade on

[12] We denote the bundle $w_a + f(a)$ usually as the *end bundle* for agent $a \in A$. It is precisely the tuple of commodities that agent $a \in A$ owns after the trade process as described by the trade–price–

some given trade–price–system. The next definition gives a very restrictive feasibility constraint.

Definition 3.3 *A* **feasible coalition** *with respect to the re-trade configuration as described by the mapping* $\Im_W : A \to 2^A$ *is a collection of agents* $F \subset A$ *such that*

$$F \subset \bigcap_{a \in F} \Im_W(a) \tag{4}$$

It is obvious that Condition 4 as formulated in Definition 3.3 is very restrictive. It states that a group of agents is feasible as a coalition in the Edgeworthian re-trading processes in the economy if everybody is directly related to each other. Within our setting it is clear that only these feasible coalitions are truly allowable to take part in the Edgeworthian recontracting processes. Given the re-trade configuration only these coalitions are truly able to coordinate their actions. Definition 3.3 therefore also reflects something about the quality of the communication as described by the mapping $\Im$. Hence, the definition of permissible coalitions above is in fact a consistency assumption on the communication possibilities as reflected in $\Im$ and the utility of that communication.

To complete the description of the Edgeworthian re-trading processes in the trade economy $\mathbf{E}$ we describe which trade–price–systems will result from these processes. As long as a certain feasible coalition is able to improve (collectively) upon a trade–price–sytem, this will be done and so this trade–price–system is unstable. This implies that for stable or *re-trade proof* trade–price–systems there is no feasible coalition, which can collectively profit from any re-trade. This is formalized in the next definition.

Definition 3.4 *A trade–price–system* $(\hat{f}, q)$ *is* **re-trade proof** *with respect to the re-trade configuration as described by* $\Im_W$*, if there is no feasible coalition* $F \subset A \setminus S_k$ *for which it holds that there exist an external re-trade function* $\hat{g} : \{(a, t(a)) \mid a \in F\} \to \mathbb{R}^\ell$ *and a re-allocation* $g : F \to \mathbb{R}^\ell$ *such that the following assertions are satisfied:*

- *For every agent* $a \in F \cap t^{-1}(F)$ *it holds that*

  $$\hat{g}(a, t(a)) = 0 \ ;$$

- *For every agent* $a \in F$

$$w_a + g(a) \geq \sum_{b \in t^{-1}(a) \setminus F} \hat{f}(b, a) \; ;$$

- *Balance in real terms:*

$$\sum_{a \in F} g(a) = \sum_{a \in F} \hat{g}(a, t(a)) \; ;$$

- *Balance in budget:*

$$\sum_{a \in F} q(\tau_a) \cdot \hat{g}(a, t(a)) \leq \sum_{a \in F} \sum_{b \in t^{-1}(a) \setminus F} q(\tau_b) \cdot \hat{f}(b, a) \; ;$$

- *For every agent $a \in F$ it holds that*

$$w_a + g(a) - \sum_{b \in t^{-1}(a) \setminus F} \hat{f}(b, a) \succ_a w_a + f(a).$$

The definition of re-trade proofness just describes inofficial or black market activities. This is illustrated by noting that all Edgeworthian re-trading or arbitraging takes place *outside* the "official" hierarchical non-cooperative trade system as described by the trade–price–system. One consequence is that any price discrimination within this system is immediately countered with some re-trade activities, provided that this is possible. Now the mapping $\Im_W$ describes these possibilities. It is therefore concluded that re-trade proofness provides another explanation for black and gray market activities as encountered a lot in eastern European economies, and also in the heavily taxed western economies.[13] With this in mind we define an equilibrium for the second stage of our model.

Definition 3.5 *An* **equilibrium** *is a re-trade proof trade–price–system $(\hat{f}, q)$ such that for $a \in S_k$ it holds that*

$$\sum_{b \in t^{-1}(a)} \hat{f}(b, a) \leq w_a,$$

and which is (strategically) individually optimal in the sense that for

[13] Another explanation is provided by a model of multilateral incentive compatibility as developed in Hammond (1987 and 1989). There it is shown that private information may lead to black or gray

$a \in S_1$: *Given the price system q, in particular $q(\tau_a)$, $w_a + f(a)$ is a maximal bundle in the budget set of a.*

$a \in A \setminus (S_1 \cup S_k)$: *Given the price $q(\tau_a)$ and the reaction curves of the agents $b \in t^{-1}(a)$, the instruments $\{q(\tau_b) \mid b \in t^{-1}(a)\}$ and $\hat{f}(\tau_a)$ are optimal in the sense that a deviation from these values of the instruments leads to* EITHER *a less preferred end bundle in the budget set* OR *to a trade–price–system which is not re-trade proof.*

$a \in S_k$: *Given the reaction curves of the agents $b \in t^{-1}(a)$, the prices $\{q(\tau_b) \mid b \in t^{-1}(a)\}$ are optimal in the sense that a deviation from these values of the instruments leads to* EITHER *a less preferred end bundle in the budget set* OR *to a trade–price–system which is not re-trade proof.*

Equilibrium in Stage II of our model is simply a trade-price-system, which is re-trade proof as well as strategically individually optimal in the sense that any agent $a \in A$ cannot improve upon it by altering his or her individual actions, taking into account the reaction curves of the other agents and the Edgeworthian re-trade processes in the economy. This is evidently conform the foundations of a hierarchical Stackelberg equilibrium as roughly introduced in the previous paragraphs as well as the Edgeworthian recontracting principle underlying many cooperative equilibrium concepts such as the Core.

3.3 Choice of trade partners

Let the trade economy $\mathbf{E} = ((A, \xi, R_\xi), \Im, g)$ be given. Then for every echelon tree $T \in \mathcal{T}_\xi$, which results from the choice of a unique predecessor per trader in $A \setminus S_k$ as described in Stage I, the previous subsection provided the definition of an equilibrium. Thus, with the use of this equilibrium concept we can evaluate every choice for a trader $a \in A \setminus S_k$, given the choices as made by the other agents in the population, i.e., in the collection $A \setminus (S_k \cup \{a\})$. This forms the foundation of our analysis of behaviour in Stage I of our model of trade.

Given $T \in \mathcal{T}_\xi$ we can denote the resulting equilibrium by the allocation $e_T \colon A \to \mathbb{R}^\ell_+$ of end bundles, where for every agent $a \in A$ we define $e_T(a) := w_a + f(a)$, with f the allocation resulting from the equilibrium trade–price–system $(\hat{f}, q)$. This implies that with every echelon tree $T \in \mathcal{T}_\xi$ there belongs a given equilibrium outcome, denoted by ϵ_T. With this simplified description we are able to define an equilibrium in the first stage of our two stage model of trade.

Firstly, we have to introduce the game that belongs to Stage I of our model of trade. This game is given by the following items:

PLAYERS: These consist of the agents $a \in A \setminus S_k$.

STRATEGIES: Every agent or player $a \in A \setminus S_k$ chooses a unique predecessor in the echelon structure (A, R_ξ), i.e., the collection of strategies available to player a is the set of predecessors $U_\xi(a) := \{b \in A \setminus S_1 \mid \{a, b\} \in R_\xi\}$.

GOALS: Every agent $a \in A \setminus S_k$ tries to achieve a maximal end bundle in the second stage of the model, given the choices of the other players in the game.

As a result of the formulation above we now define an echelon tree $T \in \mathcal{T}_\xi$ to be an **equilibrium** in the first stage of our model if it is a Nash-equilibrium in the game belonging to this stage as given above. So given the choices of *all* other agents in the population with respect to their predecessor, the agent $a \in A \setminus S_k$ chooses a unique predecessor with respect to the resulting equilibrium allocation in the second stage of the model. We can formalize this as follows:

> The echelon tree $T \in \mathcal{T}_\xi$, with predecessor function $t\colon A \setminus S_k \to A \setminus S_1$, is an equilibrium for the Stage I of the model of trade in the trade economy $\mathbf{E}$ if for every agent $a \in A \setminus S_k$ there is *no* predecessor $c \in U_\xi(a)$ such that $c \neq t(a)$ and
>
> $$\epsilon_T(a) \prec_a \epsilon_U(a),$$
>
> where $U \in \mathcal{T}_\xi$ is the echelon tree which results if agent a chooses c to be his predecessor instead of $t(a)$.[14]

Note that this Nash equilibrium is largely based on the assumptions that $\Im$ is known to all agents in the population, and that there results a unique equilibrium in the stage of actual trade after the choice of the predecessors by the agents in the economy. We note that the competition in the economy is essentially taking place in the first stage of the model.

[14]The echelon tree U is therefore determined by the predecessor function u with $u(a) = c$ and $u(b) = t(b)$ for every $b \in A \setminus (S_k \cup \{a\})$.

4 Some Re-trade Configurations

In this section we study some simple examples of trade economies by considering two different re-trade configurations. In fact we only study the equilibria that occur in the second stage of our two stage model of a trade economy as defined in Definition 3.1. This implies that essentially we study certain trade economies in which we have reduced the mapping $\Im$ to a restricted form.

In the sequel we suppose that $\mathbf{E} = ((A, \xi, R_\xi), \Im, g)$ is a trade economy, in which Stage I of our two stage trade process has already been completed. That is, every agent $a \in A \setminus S_k$ has chosen a unique predecessor, which resulted in the echelon tree $(A, W) \in \mathcal{T}_\xi$. In the following subsections we therefore only focus on the second stage of our model of trade. In these subsections we consider two cases of the mapping $\Im_W$ as a description of the Edgeworthian re-trade configuration. We subsequently discuss the cases of local re-trading, and full re-trading in the given trade economy $\mathbf{E}$.

4.1 Local re-trading

The first case we investigate in this section is that of local re-trading possibilities among agents. Basically we assume that an agent $a \in A \setminus S_k$ is able to communicate in the Edgeworthian sense with all agents, who are supplied by his or her predecessor $t(a)$.[15] Given an echelon tree $(A, W) \in \mathcal{T}_\xi$ this *local re-trade configuration* is formally described by the mapping $\Im^1_W\colon A \to 2^A$, which is given by

$$\Im^1_W(a) := \begin{cases} \{a\} \cup t^{-1}(t(a)) & \text{for} \quad a \in S_1 \\ \{a\} \cup t^{-1}(a) \cup t^{-1}(t(a)) & \text{for} \quad a \in A \setminus (S_1 \cup S_k) \\ t^{-1}(a) & \text{for} \quad a \in S_k \end{cases}$$

It may be clear that the re-trade configuration $\Im^1_W$ describes a situation in which the agents know the prices which are charged by their predecessor to themselves as well as to the other successors of that predecessor. First we deal with the consequences of the assumptions above with respect to the re-trade proofness of a trade–price–system in this setting.

Therefore we note that a group of agents $F \subset A$ is a feasible coalition with respect to the re-trade configuration $\Im^1_W$ if and only if there is an agent $a \in A \setminus S_1$

[15] Thus, in principle for all agents $a \in A \setminus S_k$ it holds that $t^{-1}(t(a)) \subset \Im_W(a)$, where $\Im_W$ is the re-trade configuration.

such that $F \subset t^{-1}(a)$. So any feasible coalition consists of agents which are supplied by the *same* predecessor.

The obvious consequence of re-trade proofness is that there cannot be any local price discrimination[16], i.e., any agent $a \in A \setminus S_1$ has to charge the same prices for his or her successors $b \in t^{-1}(a)$. This involves a reduction of the instruments, which are available to some agent $a \in A \setminus S_1$. This leads to the following equilibrium.

$a \in S_1$:

It is evident that the net-trade bundle on the activated trade relation $\tau_a \in W$ is the only instrument which is available, while the predecessor $t(a)$ sets a price $q_{t(a)}$ on that active trade relation. This results in the following budget set:

$$B_a := \left\{ w_a + d \;\middle|\; \begin{array}{l} q_{t(a)} \cdot d \leq 0 \\ w_a + d \geq 0 \end{array} \right\} \subset \mathbf{R}_+^\ell.$$

Now an agent a chooses his net-trade bundle d_a such that

$$w_a + d_a \in \max_{\succ_a} \; B_a.$$

This individually rational and optimal choice is now denoted by $d_a(q_{t(a)})$.

$a \in A \setminus (S_1 \cup S_k)$:

With the reduced instruments available we can describe the behaviour of such an intermediairy agent as follows: His or her predecessor $t(a)$ sets a price $q_{t(a)}$ on the trade relation τ_a, while agent a determines the net-trade d on this trade relation. Moreover, agent a chooses a unique price vector $\hat{q} \in S^\ell$ on all trade relations $\{\tau_b \mid b \in t^{-1}(a)\}$. Finally, agent a takes into account the demand functions of every successor $b \in t^{-1}(a)$, which we denote by $d_b(\cdot)$, and which is the individually optimal response of agent b to the chosen price vector $\hat{q}$ by a. This concludes to the following definition of the budget set of agent a:

$$B_a(\hat{q}) := \left\{ w_a + d - \sum d_b(\hat{q}) \;\middle|\; \begin{array}{l} q_{t(a)} \cdot d \leq \hat{q} \cdot \sum d_b(\hat{q}) \\ w_a + d \geq \sum d_b(\hat{q}) \end{array} \right\} \subset \mathbf{R}_+^\ell.$$

[16]From the assumptions it is clear that all agents in the economy have monotonuous preferences. As a consequence, any price discrimination between members of some *feasible* coalition immediately leads to re-trading within this coalition. Hence, re-trade proofness requires that within a feasible coalition there cannot be price discrimination.

Now the agent a chooses his net-trade bundle d_a and the price vector $q_a \in S^\ell$ such that

$$w_a + d_a - \sum d_b(q_a) \in \max_{\succ_a} B_a(q_a).$$

This optimal choice is denoted by $d_a(q_{t(a)})$ and q_a.

$a \in S_k$:

This unique agent has no predecessor and, hence, has no vector of net-trades as his or her instrument. This leaves this agent with the determination of prices on all relations (b, a) with $b \in t^{-1}(a)$. Given the definition of re-trade proofness and the re-trade configuration these prices have to be chosen uniform. Thus, the agent $a \in S_k$ chooses a unique price vector $\hat{q}$ on the trade relations with his or her successors. Of course the agent takes into account the optimal responses of his or her successors to changes in the price vector. For his or her budget set we therefore get the following expression:

$$B_a(\hat{q}) := \left\{ w_a - \sum d_b(\hat{q}) \;\middle|\; \begin{array}{l} 0 \leq \hat{q} \cdot \sum d_b(\hat{q}) \\ \\ w_a \geq \sum d_b(\hat{q}) \end{array} \right\} \subset \mathbb{R}^\ell_+.$$

Now the agent chooses the unique price vectors to be $q_a \in S^\ell$ such that

$$w_a - \sum d_b(q_a) \in \max_{\succ_a} B_a(q_a).$$

This optimal choice is denoted by q_a.

We are able to formulate an equilibrium trade–price–system with respect to the given re-trade configuration $\Im^1_W$. This is done in the next result.

Theorem 4.1 *The trade-price system $(\hat{f}, q)$, which for every activated trade relation $\tau_a \in W$ is given by*

$$q(\tau_a) = q_{t(a)} \quad \textit{and}$$

$$\hat{f}(\tau_a) = d_a(q_{t(a)}),$$

is an equilibrium in the trade economy $\mathbf{E} = ((A, \xi, R_\xi), \Im^1, g)$ endowed with the echelon tree $(A, W) \in \mathcal{T}_\xi$.

PROOF

The trade–price–system $(\hat{f}, q)$ as defined above is evidently individually optimal. Moreover, by construction it satisfies the feasibility condition of the top trader $a \in S_k$. We therefore only have to check whether this trade–price–system is re-trade proof. The argument, which we use, is standard in core theory. Suppose that there exists a feasible coalition $F \subset A$ which can improve upon this trade–price–system by some net trade function $\hat{g}: \{(a, t(a)) \mid a \in F\} \to \mathbf{R}^\ell$ and some re-allocation $g: F \to \mathbf{R}^\ell$, which satisfy the conditions as given in Definition 3.4.

Since F is a feasible coalition it follows that there exists an agent $c \in A \setminus S_1$ such that $F \subset t^{-1}(c)$. From the definition of the trade–price–system it is evident that c charges the same price, say q_c, to all successors in $t^{-1}(c)$, and hence to all agents $a \in F$. Moreover we know that $\hat{f}$ is an optimal net-trade in the budget set for agent $a \in F$. Since $\hat{g}$ gives a strictly better allocation, it has to follow that for every $a \in F$ it holds that

$$q_c \cdot g_a > \sum_{b \in t^{-1}(a)} q(\tau_b) \cdot \hat{f}(b, a).$$

Now from the "balance in real terms" condition as formulated in Definition 3.4 it is evident that:

$$\begin{aligned} \sum_{a \in F} q_c \cdot \hat{g}_a &= q_c \cdot \sum_{a \in F} \hat{g}_a = q_c \cdot \sum_{a \in F} g_a = \\ &= \sum_{a \in F} q_c \cdot g_a > \sum_{a \in F} \sum_{b \in t^{-1}(a)} q(\tau_b) \cdot \hat{f}(b, a). \end{aligned}$$

This is in contradiction with the "balance in budget" condition on the net-trades $\hat{g}$ as formulated in Definition 3.4. So we may conclude that the individually optimal trade–price–system $(\hat{f}, q)$ as defined above is in fact re-trade proof, and hence is an equilibrium.

Q.E.D.

The equilibrium as defined above is one without local price discrimination, but does not exclude global price discrimination. We conclude that with local information there still may be price differences due to different (informational) positions in the hierarchical trade structure of the economy. The local information assumption can however be regarded as a sustainable assumption. It is quite normal that an agent is informed on the prices which are charged by his or her predecessor, also to the other successors.

4.2 Full re-trade configurations

In this subsection we investigate under which conditions on the re-trade configuration of the economy, the resulting equilibrium in the second stage of our model is Walrasian. The main result is that certain re-trade configurations, which we denote as *full*, lead to uniform pricing, and hence to a Walrasian equilibrium. The main lesson of this result is that the structure of the market is, besides strategic behaviour and largeness of the market, a fundamental tool in supporting Walrasian equilibria.

Definition 4.2 *Let* $\mathbf{E} = ((A, \xi, R_\xi), \Im, g)$ *be a trade economy. The re-trade configuration* $\Im\colon \mathcal{T}_\xi \times A \to 2^A$ *is* **full** *with respect to the echelon tree* $(A, W) \in \mathcal{T}_\xi$ *if there exists an ordering of the set of agents* $A \setminus S_k$*, denoted by* $\{a_1, \ldots, a_{n-1}\}$ *with* $n = \# A$*, such that the following properties are satisfied:*

1. *For every* $i \in \{1, \ldots, n-2\}$ *it holds that* $a_{i+1} \notin t^{-1}(a_i)$ *as well as* $a_{i+1} \neq t(a_i)$*.*
2. *For every* $i \in \{2, \ldots, n-2\}$ *it holds that*
$$\{a_{i-1}, a_i, a_{i+1}\} \subset \Im_W(a_i).$$
3. $$\{a_{n-2}, a_{n-1}\} \subset \Im_W(a_{n-1}).$$
4. $$\{a_1, a_2\} \subset \Im_W(a_1).$$

In order to describe the properties of an equilibrium resulting in case of a full re-trade configuration, we introduce the notion of uniform pricing. More formally, let $\mathbf{E}$ be a trade economy, and let $(A, W) \in \mathcal{T}_\xi$ be an echelon tree. Then we define a **uniform pricing trade–price–system** as a trade–price–system $(\hat{f}, q)$, where for every agent $a \in A \setminus S_k$ it holds that $q(\tau_a) = \hat{q}$ and $\hat{f}(\tau_a) = d_a(\hat{q})$, where $\hat{q}$ and the demand functions d_a are determined by the following rules:

$\hat{a} \in S_k = \{\hat{a}\}$:

This agent determines the value of the price vector $\hat{q}$ given the rule that he or she whishes to reach an optimal end-bundle in his or her budget set. We note that this budget set is given by

$$B_a(\tilde{q}) := \left\{ \; w_a - \sum d_b(\tilde{q}) \;\middle|\; \begin{array}{l} 0 \leq \tilde{q} \cdot \sum d_b(\tilde{q}) \\ \\ w_a \geq \sum d_b(\tilde{q}) \end{array} \right\} \subset \mathbb{R}^\ell_+.$$

Now the agent chooses the unique price vector $\hat{q} \in S^\ell$ to be such that

$$w_a - \sum d_b(\hat{q}) \in \max_{\succ_a} B_a(\hat{q}).$$

$a \in A \setminus (S_1 \cup S_k)$:

These agents act as price takers with respect to the price $\hat{q}$. Thus on the trade relations with their successors they set this price vector $\hat{q} \in S^\ell$. This determines their budget set as follows:

$$B_a(\hat{q}) := \left\{ w_a + d - \sum d_b(\hat{q}) \;\middle|\; \begin{array}{l} \hat{q} \cdot d \leq \hat{q} \cdot \sum d_b(\hat{q}) \\ \\ w_a + d \geq \sum d_b(\hat{q}) \end{array} \right\} \subset \mathbf{R}^\ell_+.$$

Since the agent a wants to achieve an optimal end-bundle, given the budget set, he or she chooses the net-trade bundle d_a such that

$$w_a + d_a - \sum d_b(\hat{q}) \in \max_{\succ_a} B_a(\hat{q}).$$

This determines the demand bundle $d_a(\hat{q})$ for the agents $a \in A \setminus (S_1 \cup S_k)$.

$a \in S_1$:

These agents act as price takers with respect to the price quoted on their price relation. In case of uniform pricing this quoted price vector is $\hat{q}$. Thus the budget set becomes

$$B_a(\hat{q}) = \left\{ w_a + d \;\middle|\; \begin{array}{c} \hat{q} \cdot d \leq 0 \\ \\ w_a + d \geq 0 \end{array} \right\} \subset \mathbf{R}^\ell_+.$$

Since the agent a wants to achieve an optimal end-bundle, he or she chooses the net-trade bundle d_a such that

$$w_a + d_a \in \max_{\succ_a} B_a.$$

This individually rational and optimal choice is denoted by $d_a(\hat{q})$.

The description above completes the characterization of a uniform pricing trade–price–system. We note that there may exist more than one of such uniform pricing trade–price–systems.

Immediately we may conclude that a uniform pricing trade-price-system is indeed re-trade proof with respect to any re-trade configuration, and hence with respect to a full re-trade configuration. It does however not have to be an equilibrium in the second stage of our model of trade, since it does not have to be individually optimal with respect to the pricing rules. The next theorem states that if it is indeed an equilibrium for the second stage of our model, then it has to be a Walrasian equilibrium.

Theorem 4.3 *Let* $\mathbf{E}$ *be a trade economy such that a uniform pricing trade-price-system is an equilibrium for the second stage in case the echelon tree* $(A, W) \in \mathcal{T}_\xi$ *occurs. If* $w_{\tilde{a}} = 0$, *with* $\tilde{a} \in S_k$, *then this equilibrium is Walrasian in which the agent* $\tilde{a}$ *plays the rôle of auctioneer.*

PROOF
Suppose that $a \in A \setminus S_k$, i.e., $a \neq \tilde{a}$. Then we define

$$\hat{d}_a(\hat{q}) := d_a(\hat{q}) - \sum_{b \in \iota^{-1}(a)} d_b(\hat{q}).$$

Now note that from the definition of a uniform pricing trade-price-system, that $\hat{d}_a$ is chosen such that

$$w_a + \hat{d}_a \in \max_{\succ_a} \left\{ w_a + d \;\middle|\; \begin{array}{c} \hat{q} \cdot d \leq 0 \\ \\ w_a + d \geq 0 \end{array} \right\}.$$

This is equal to the normal budget set in a finite pure exchange economy at price $\hat{q}$. So we conclude that $\hat{d}_a(\hat{q})$ is the Walrasian net-demand at price $\hat{q}$. We next show that the price $\hat{q}$ is in fact the Walrasian equilibrium price. Firstly we note that

$$\sum_{a \neq \tilde{a}} \hat{d}_a(q) = \sum_{a \neq \tilde{a}} d_a(q) - \sum_{a \neq \tilde{a}} \sum_{\iota^{-1}(a)} d_b(q) = \sum_{\iota^{-1}(\tilde{a})} d_b(q). \tag{5}$$

From Equation 5 the budget constraint of the agent $\tilde{a}$ reduces to the normal law of Walras, i.e., $q \cdot \sum_{a \neq \tilde{a}} \hat{d}_a(q) = 0$. Moreover, the feasibility constraint of that agent reduces to

$$w_{\tilde{a}} \geq \sum_{\iota^{-1}(\tilde{a})} d_b(q) = \sum_{a \neq \tilde{a}} \hat{d}_a(q).$$

So if, as assumed, $w_{\tilde{a}} = 0$, then we simply get the market clearing condition:

$$\sum_{a \neq \tilde{a}} \hat{d}_a(\hat{q}) \leq 0.$$

Together with the assumption that the preference of agent $\tilde{a}$ is monotonuous, we arrive at the conclusion that the price $\hat{q}$ has to be a Walrasian market clearing price.

Q.E.D.

The next result concludes the analysis of full re-trade configurations and their connection with Walrasian equilibria. It states that in case the re-trade configuration is full, the uniform pricing trade–price–system is supported as an equilibrium for the second stage of our model of trade. Together with Theorem 4.3 this shows that in case the re-trade configuration is full and the top trader $\tilde{a} \in S_k$ has no endowment, our model supports the Walrasian equilibrium.

Theorem 4.4 *Let* $\mathbf{E}$ *be a trade economy, and let* $(A, W) \in \mathcal{T}_\xi$ *be an echelon tree. If the re-trade configuration* $\Im$ *is full with respect to* (A, W)*, then the equilibrium for the second stage of our model of trade is a uniform pricing trade–price–system.*

PROOF

It is immediately clear that if a uniform pricing trade–price–system is individually optimal with respect to the pricing rules, taking into account its re-trade proofness, we have shown that it is indeed an equilibrium. It therefore suffices to show that re-trading in the setting as described in the assertion leads to uniform pricing, i.e., for any re-trade proof trade–price–system $(\hat{f}, q)$ it holds that for every trade relation $\tau \in W$: $q(\tau) = \hat{q}$ with $\hat{q} \in S^\ell$. This supports the assertion that, given the re-trade configuration, uniform pricing is indeed individually optimal.

Let $\{a_1, \ldots, a_{n-1}\}$ be the ordered set of agents in the sense of the definition of a full re-trade configuration. Now assume that there exists a re-trade proof trade–price–system without uniform pricing. Thus, there exist $i, j \in \{1, \ldots, n-1\}$ with $i < j$ such that $q(\tau_{a_i}) \neq q(\tau_{a_j})$.

Then by the properties of the ordering it is obvious that there has to be an index $i \leq h \leq j-1$ such that $q(\tau_{a_h}) \neq q(\tau_{a_{h+1}})$. But by definition the coalition $F = \{h, h+1\}$ is feasible with respect to the re-trade configuration, and hence can exploit the price differences as quoted on their trade relations. This is in contradiction with our assumption that the trade–price–system $(\hat{f}, q)$ is re-trade proof.

Q.E.D.

5 Concluding Remarks

The model as developed in this paper is just a single example how to deal with economic decision mechanisms in the setting of limited trade possibilities among the agents in the economy. We explicitly remark that there are more possibilities in describing these specific decision processes. In this section we discuss some remarks with respect to the two stage model of trade as sketched and analysed in this paper.

Our first remark focusses on the problem of existence of the two stage equilibrium concept as sketched in this paper. It is our conjecture that there are some severe problems to be solved with respect to this question after its existence. Especially the consequences of the implementation of an arbitrary re-trade configuration as described by the mapping $\Im$ may lead us to the conclusion that the existence problem has to be solved for *any* such configuration separately. It may also be clear that the two cases as considered in Section 4 do not give many severe problems with respect to existence. The problem of the uniqueness of the optimal strategy, i.e., the net-trade bundle, for every trader in the collection $A \setminus S_k$ may lead to some strong assumptions, but can probably be resolved. In less trivial cases as those sketched in Section 4 there may be some problems with respect to the existence of the equilibrium.

Another angle to approach the problem of existence is the possibility to apply the theory of social systems with coordination as developed by Vind (1983 and 1986). The main tool in this approach is the general existence theorem as provided by Keiding (1985). The existence problem will be addressed in a subsequent paper dealing with the specific case of a minimal re-trade configuration, i.e., $\Im$ is such that it satisfies exactly the minimal requirements as described in its definition.

The main issue as addressed in this paper is not only the design of an equilibrium concept dealing with limited trade possibilities among the agents in the economy, but also the analysis which cooperative Edgeworthian re-trade possibilities would force this equilibrium to be Walrasian. The main results in Section 4 show that the market organization structure, especially with respect to cooperative trade, can force the market equilibrium to be a uniform pricing equilibrium and even a Walrasian equilibrium.

This shows that we have an important additional feature, which supports perfectly competitive or price taking behaviour of agents on the market. In the literature one deals with large markets as the natural environment of perfectly competitive behaviour. Besides that, research has shown that in certain cases of *small* markets, perfectly competitive behaviour can be supported as strategically optimal. The results in this paper show that in certain cases of small markets, perfect competition can also be sustained by the organization of the market and the cooperative re-trading possibilities of the agents on the market.

This also supports our conclusion that the structure of the market or the organization of trade activities are important issues to be addressed in future research.

Finally we note that we can link our model with models of oligopolistic markets. This is done with the use of a specific re-trade configuration. Let $\xi \in \mathcal{H}$ with length $k = \ell(\xi)$, then for some echelon tree $(A, W) \in \mathcal{T}_\xi$ with predecessor function $t\colon A \setminus S_k \to A \setminus S_1$ the re-trade configuration is given by

$$\Im_W(a) := t(S_j),$$

for every agent $a \in S_j$ with $1 \leq j \leq k-1$. Now the collection of all feasible coalitions is just described by the collection of coalitions $E \subset S_j$ for some $1 \leq j \leq k-1$. This implies that re-trade proofness requires that there is uniform pricing on all trade relations in $(S_j \times N_{j+1}) \cap W$. Hence there is uniform pricing on all agents in the echelon S_j.

This means that there is "perfect competition" between agents in N_{j+1} supplying agents in this particular echelon. With the use of this re-trade configuration we are describing a situation of a hierarchy of perfectly competitive markets with a finite number of suppliers. This leads to the conclusion that full information on pricing may annihilate the effect of being "large". We however note that we only consider pure exchange in our models. In those cases uniform pricing is one of the major benchmarks on the perfectness of competition on the markets. Especially for models in which pricing is the main strategy, as is the case in the model as described in the previous section. In that case we may even refer to that model as a *Bertrand–competitive* model of oligopolistic markets with limited trade possibilities.[17]

To complete our remarks with respect to links with the theory on oligopolistic markets, we make some comments and suggestions with respect to the re-trade configurations to be chosen. By introducing uncertainty with respect to the information, which is available to the agent and as reflected in the mapping $\Im_W$, we can

[17]Note that we deal with homogeneous commodities in the case of a pure exchange economy.

describe limited possibilities of re-trading. The set $\Im_W(a) \subset A$ of an agent $a \in A$ therefore gets a fuzzy character. This fuzzyness leads to limited commitment to a single predecessor in the trade structure. The model may be conformable with the approach as described in Allen and Thisse (1988), where a measure is given on the set of consumers, describing the marketsize of the oligopolistic suppliers on the market. In our hierarchical model this can be generalized by defining such measures for every market in the hierarchy. The most important step in the model would be to give a foundation to these measures with the use of the fuzzy re-trade configuration.

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